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EMISSIVE VERSUS ATTENUATING SMOKES

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Emissive smoke clouds reduce visibility by attenuating the image that is being viewed along with superimposing scattered and emitted radiation onto that image. Thus, contrast and signal to noise ratio are reduced by a combination of attenuation (governed by the extinction coefficient), scatter (governed by the single scatter albedo), and emission (governed by single scatter albedo and cloud temperature). Two alternate approaches are investigated. First, the entire cloud including air is heated. Secondly, flare particles are placed within a scattering smoke. The first approach is found to be impractical because of energy requirements and cloud buoyancy while the second approach appears to significantly reduce contrast and signal to noise.

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PREFACE

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CONTENTS

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EMISSIVE VERSUS ATTENUATING SMOKES

1.   INTRODUCTION

        Theory is developed and one dimensional radiative transfer calculations are carried out predicting contrast and signal to noise transmittances of two types of smoke with the purpose of determining what reductions in optical depth would be possible when smoke thermal emission is increased. First, analysis was completed on scattering, absorbing and emitting smoke clouds at elevated temperatures where entrained air temperature is equal to aerosol particle temperature. Second, analysis was completed on smoke clouds containing both scattering nonabsorbing smoke particles at ambient temperature and high temperature emitting flare particles. In the first case it was found that heating the smoke particles will not significantly reduce contrast transmittance or signal to noise ratio transmittance because smoke particles dump heat into the air within a few microseconds and even the most exothermic reactions are not energetic enough to heat the entrained air mass more than a few degrees Kelvin at smoke battlefield concentrations of a few tenths of a gram of aerosol per cubic meter of air. If the heat comes from another source such as smoke particles injected into a hot stream of engine exhaust gas or pyrotechnically produced gas, the cloud temperature would have to be elevated to a level where it would become buoyant and rise rapidly before significant reductions in optical depth could be realized. In the second case, if flare particles are much larger than the smoke particles they will dissipate heat primarily by radiation rather than conduction into air and much higher temperatures and thermal emission can be achieved. On the other hand when flare particles are made larger their emission cross section per mass decreases and the gravitational settling velocity increases. Therefore flare particle size, shape and reaction rate or temperature must be optimized to balance maximization of flare particle emission and duration with minimization of attenuating particle cloud optical depth maintaining contrast and signal to noise ratio transmittance at an acceptable level. The optical depth requirement can be interpreted as a requirement on either extinction coefficient or concentration pathlength product since the optical depth is the product of both. Calculations based on one dimensional radiative transfer are most accurate for extensive smoke blankets and walls where photon flux passing through cloud edges can be neglected.

2.   CONTRAST

        The ability to see an object depends on the contrast between the object and background.[1] When smoke is placed between the observer and object the contrast is affected in two ways; first the smoke attenuates the radiance of the object and background, second the smoke superimposes scattered and emitted radiance onto the scene. Smoke attenuation of background and object radiance is predicted by the smoke's unscattered transmittance while radiance superimposed onto the scene is predicted by summing contributions due to reflected, scattered transmitted and emitted radiance. In order to quantify the effects of attenuation vs scatter plus emission on contrast we first recall a standard definition of contrast of an object against a background[2]

$$C = (L_o - L_b)/(L_o + L_b)$$

where L_o is the object radiance and L_b his background radiant and then write the contrast of that object against that background viewed through smoke

$$C' = (L_o' - L_b')/(L_o' + L_b')$$

where the object and background radiances viewed through smoke are

$$L_o' = T_u L_o + T_s L_+ + RL_- + E\sigma T^4 \quad \text{and} \quad L_b' = T_u L_b + T_s L_+ + RL_- + E\sigma T^4$$

Here T_u is the unscattered (Beer's Law) transmittance, T_s is the scattered light transmittance, R is the reflectance, L_- is ambient sky plus solar plus background terrain radiance directed away from the observer, L_+ is ambient sky plus solar plus background terrain radiance directed toward the observer, E is the smoke emitted radiance, σ is the Stefan Boltzman constant and T is the cloud temperature. The unscattered transmittance through the smoke depends only on the extinction coefficient, extinction cross section per mass of smoke, and the concentration path length product. The smoke reflectance and scattered light transmittance depend on these parameters as well as single scatter albedo, the ratio of scatter cross section to extinction cross section and phase function (the differential scattering pattern). Smoke emission depends on all of the previously mentioned parameters as well as the cloud temperature in the case of thermal emission and other properties in the case of fluorescence.

3.   CONTRAST TRANSMITTANCE AND ONE DIMENSIONAL RADIATIVE TRANSFER MODEL

To quickly analyze the effects of all these smoke properties upon contrast it is convenient to work with a one dimensional radiative transfer model. Here the smoke is considered as a smoke wall or smoke blanket and the analysis requires considerably less computation time due to the simple closed form nature of these solutions rather than the numerical solutions involved with three-dimensional radiative transfer. In this way we dispense with considerations of cloud shape and transport of radiation perpendicular to the line of sight, non uniform backgrounds and non uniform ambient illumination. Further simplifications that speed computation involve replacing the phase function with a single number, the asymmetry factor and reducing the flow of radiation to diffuse and collimated components moving in both directions by using the four flux model of one-dimensional radiative transfer where the flux is considered as a combination of incident collimated flux passing through the smoke layer in both directions along with scattered flux going in both directions. Diffuse incident flux can be represented by averaging collimated flux over angle.

Two simplifications result when we work with the contrast transmittance C_T, ratio of contrast observed through smoke over contrast observed without smoke present,

$$C_T = \left(\frac{T_u(L_o - L_b)}{T_u(L_o + L_b) + 2T_s L_+ + 2RL_- + 2E\sigma T^4} \right)\left(\frac{L_o + L_b}{L_o - L_b} \right) = \frac{1}{1 + \dfrac{2T_s L_+ + 2RL_- + 2E\sigma T^4}{T_u(L_o + L_b)}}$$

rather than just contrast. First the range of C_T always lies between 0 and 1. Second the set of four radiance ratios L_b/L_o, L_+/L_o, L_-/L_o and E/L_o reduce to three; $L_+/(L_o+L_b)$, $L_-/(L_o+L_b)$ and $E/(L_o+L_b)$. The four flux theory one dimensional radiative transfer theory connects the smoke blanket or smoke wall unscattered light transmittance T_u, reflectance R, scattered light transmittance T_s and emittance E with smoke particle properties, aerosol concentration and cloud pathlength[3]

$$T_u = e^{-b/\mu}$$

$$R = q\left(e^{zb/\mu} - e^{-zb/\mu} \right) \Big/ \left((1-p+z)e^{zb/\mu} - (1-p-z)e^{-zb/\mu} \right)$$

8

$$T_s = \frac{2\,z}{(1-p+z)e^{zb/\mu} - (1-p-z)e^{-zb/\mu}} - e^{-b/\mu}$$

$$E = (1 - R - T_s - T_u)$$

where

$$p = \frac{a}{2}(1+g)$$

$$q = \frac{a}{2}(1-g)$$

$$z = \left((1-a)(1-ag)\right)^{1/2}.$$

The smoke optical depth is designated as b, single scatter albedo is designated as a, asymmetry factor is g, and μ is the cosine of the angle of incidence with respect to the cloud wall or blanket surface.

4. SIGNAL TO NOISE RATIO AND SIGNAL TO NOISE TRANSMITTANCE

For photon noise limited detection a signal to noise ratio SN is defined in terms of the number of photons collected n which is proportional to the collected radiance L

$$n = \beta L$$

where β is the proportionality constant incorporating the collection area and time.[4,5] Setting the noise equal to the square root of the number of photons collected in each measurement and the total noise of object and background measurements equal to the square root of the sum of the squares we have

$$SN = \frac{\beta\left[\left(T_u L_o + T_s L_+ + R L_- + E\sigma T^4\right) - \left(T_u L_b + T_s L_+ + R L_- + E\sigma T^4\right)\right]}{\sqrt{\beta\left[\left(T_u L_o + T_s L_+ + R L_- + E\sigma T^4\right) + \left(T_u L_b + T_s L_+ + R L_- + E\sigma T^4\right)\right]}}$$

$$= \sqrt{\beta}\,T_u\left(L_o - L_b\right) \Big/ \left(T_u\left(L_o + L_b\right) + 2T_s L_+ + 2R L_- + 2E\sigma T^4\right)^{1/2}$$

We define signal to noise transmittance SN_T as the ratio of signal to noise ratio looking through smoke divided by the signal to noise ratio without smoke present.

$$SN_T = \frac{\sqrt{\beta}\,T_u\left(L_o - L_b\right)}{\left(T_u\left(L_o + L_b\right) + 2T_s L_+ + 2R L_- + 2E\sigma T^4\right)} \frac{\left(L_o + L_b\right)}{\sqrt{\beta}\left(L_o - L_b\right)}$$

9

$$SN_T = \cfrac{1}{\left(1 + \cfrac{2T_sL_+}{T_u(L_o+L_b)} + \cfrac{2RL_-}{T_u(L_o+L_b)} + \cfrac{2E\sigma T^4}{T_u(L_o+L_b)}\right)^{1/2}} = \cfrac{T_u^{1/2}}{\left(1 + \cfrac{2T_sL_+ + 2RL_- + 2E\sigma T^4}{T_u(L_o+L_b)}\right)^{1/2}}$$

Contrast transmittance is simply related to signal to noise transmittance

$$SN_T = \left(T_u C_T\right)^{1/2}$$

Three simplifications result when we work with signal to noise transmittance instead of the signal to noise ratio. First, the range of SN_T lies between 0 and 1. Second, the set of four radiance ratios L_b/L_0, L_+/L_0, L_-/L_0 and E/L_0 reduce to three; $L_+/(L_o+L_b)$, $L_-/(L_o+L_b)$ and $E/(L_o+L_b)$. Finally, the proportionality constant β can be eliminated.

It is clear from these expressions that contrast transmittance and signal to noise transmittance are reduced by increasing cloud reflectance and scattered light transmittance but they are also reduced by increasing cloud emittance and temperature. Two parameters are defined to quantify decreases in contrast and signal to noise transmittance due to heating the cloud relative to those decreases produced by an identical unheated cloud. These parameters are the contrast transmittance ratio CTR and the signal to noise transmittance ratio SNTR; the heated cloud value divided by the unheated cloud value.

5.   HEATING SMOKE PARTICLES BY REACTIONS AND EXTERNAL MEANS

Heating smoke particles by an exothermic reaction or using external radiation from an energetic beam such as a high energy laser will quickly lead to heat loss to the air mass within the smoke cloud which is generally about four orders of magnitude greater than the aerosol mass within the cloud. Exothermic smoke reactions producing ΔH joules/gram of smoke particles will quickly elevate smoke cloud temperature by ΔT by

$$\Delta T = \Delta H \, C / (\rho \, c)$$

where C is smoke concentration in grams/m³, ρ is air density (~1,300 g/m³) and c is the heat capacity of air (~1 j/g°K). In order to raise smoke cloud temperature by more than ΔT a reaction must produce ΔH greater than

$$\Delta H = \rho \, c \, \Delta T / C$$

For example, a smoke particle reaction would have to produce greater than 20,000 J/g of heat to raise cloud temperature more than 3°K at smoke concentrations under 0.2 g/m³. Similarly we can heat the aerosol cloud using an external source of radiation such as a high energy laser or spotlight. The temperature rise depends on the absorbed radiance in accordance with the energy balance equation

$$T_u \, E \, \alpha_a C \, A \, dl = c \, \rho \, \Delta T \, A \, dl$$

where T_u is the unscattered light transmittance, E is the incident high energy density in joules/m², α_a is the aerosol absorption coefficient (absorption cross section per aerosol mass), A is the cloud area transverse to radiance propagation and dl is the incremental pathlength in the cloud along the beam. Thus to raise smoke temperature by more than ΔT the high energy density must exceed

10

$$E > c \, \rho \, \Delta T \, / \, (\, T_u \, \alpha_a \, C \,)$$

For example, incident energy densities greater than 19,500 J/m^2, achievable with a one meter diameter 19.5 kilowatt beam incident on a cloud in a 1 meter per second cross wind, would be required to raise the temperature of the cloud edge facing the incident power by 3°K for a cloud with an absorption coefficient of 1 m^2/g at a concentration of 0.2 g/m^3. Finally a third approach would involve heating the aerosol and air by mixing with a hot exhaust gas stream or pyrotechnically generated hot gas flow. Now the external power P that is required to elevate the temperature by ΔT is given by

$$P = c \dot{m} \Delta T$$

where $\dot{m}$ is the mass flow rate of hot exhaust or pyrotechnically generated gas plus aerosol plus entrained ambient air that is mixed producing the smoke cloud. For example, 6 kilowatts of power must be provided to elevate a mass flow rate of 2 kilograms per second by 3 °K.

6.   HEATED SMOKE CLOUD BUOYANCY

Heated clouds will of course rise due to their buoyancy. The velocity of rise can be estimated by balancing the buoyancy force acting on the cloud

$$F_B = \Delta \rho \, (\pi/6) \, D^3 \, g$$

with the drag force acting on the cloud

$$F_D = C_D \, (\pi/4) \, D^2 \, \rho \, v^2 \, / \, 2$$

where $\Delta \rho$ is the reduction in cloud density due to heating, D is the cloud diameter, g is acceleration due to gravity (~10m/s^2), C_D is the air drag coefficient acting on the entire cloud and v is the velocity that the cloud is rising. The ideal gas law predicts the reduction in cloud density due to a temperature elevation ΔT above ambient temperature T

$$\Delta \rho = \rho / \, (\, 1 + T \, / \, \Delta T \,)$$

The drag coefficient for Reynolds numbers greater than 200,000 is 0.1. A correction factor that reduces this drag coefficient due to internal air flow within the cloud can be computed in the laminar flow regime and gives the value 5/6 for a gas sphere moving through a gas of equal viscosity.[6] Although we are outside the laminar flow regime we can expect the cloud to break up anyway once velocities become great enough to produce significant turbulence. The Reynolds number Re is given by

$$Re = v \, D \, / \, \mu$$

where μ is the kinematic viscosity of air (0.152 cm^2/s at 293°K). Setting the buoyancy force equal to the air drag force and solving for the velocity of cloud rise we have

$$v = (\, (\, \Delta \rho \, (\pi/6) D^3 \, g \,) \, / \, (\, C_D \, (\pi/4) D^2 \, \rho \, / \, 2 \,) \,)^{1/2} = (\, 4 \, \Delta \rho \, g \, D \, / \, (\, 3 \, \rho \, C_D \,))^{1/2}$$

Substituting for $\Delta \rho$ we get

$$v = \sqrt{\frac{4gD}{3\left(1+\dfrac{T}{\Delta T}\right)C_D}}$$

For example, a 10 meter diameter cloud with a temperature elevation of 3°K above an ambient temperature of 300°K would rise at a velocity of 3.6 m/s. This corresponds to a Reynolds number

$$Re = (3.6 \text{ m/s}) (10 \text{ m}) / (0.0000152 \text{ m}^2/\text{s}) \sim 2,370,000$$

which is in the range where the drag coefficient is approximately equal to 0.1 but we can expect turbulent flow to quickly break apart the cloud in this flow regime.

7.   TIME SCALE FOR HEAT TRANSFER FROM AEROSOL PARTICLES

The rate of heat conduction away from the smoke particles into the air surrounding the particles can be estimated using the instantaneous or continuous point heat source solutions for transient heat conduction into an infinite medium. The instantaneous point source solution is[7]

$$\Delta T(r,t) = \frac{Q}{8(\pi\kappa t)^{3/2}\rho c} e^{-\frac{r^2}{4\kappa t}}$$

where κ is the thermal diffusivity of air, 0.187 cm²/s, Q is the quantity of heat instantaneously released at time t = 0 and r is the distance from the point source. The temperature has its maximum value at $t = r^2/(6\kappa)$ which is also the time at which the heat has reached a mean square distance r^2 from the instantaneous source. The continuous point source solution is[7]

$$\Delta T(r,t) = \frac{\dot{Q}}{4\pi\kappa r\rho c} erfc\frac{r}{\sqrt{4\kappa t}}$$

where $\dot{Q}$ is the rate of release by the continuous point source and *erfc* is the complimentary error function which can be approximated by the expression[8]

$$erfc(x) = 1 - \sqrt{1 - e^{-\frac{4x^2}{\pi}}}$$

After a time $t = r^2/(6\kappa)$ the heat initially released by the continuous source has moved a mean square distance r^2 away from the source and $erfc((3/2)^{1/2}) = 0.077$, which is still a small fraction of the steady state solution

$$\Delta T(r, t = \infty) = \frac{\dot{Q}}{4\pi\kappa r\rho c}$$

It is important to recognize that the instantaneous, continuous and steady state point source solutions apply to ensembles of particles up to times where there is insignificant heating of an air parcel by more distant particles relative to heating by the nearest particle. In order to determine how quickly an aerosol particle dumps heat into the air we compute the time required for a particle to heat air

12

within the cloud located a maximum distance from any smoke particle. Therefore we must estimate interparticle spacing within a cloud. For typical smoke clouds we assume a particle volume of one cubic micrometer and a typical concentration of $0.2g/m^3$. If we assume a typical particle density of 2 then each particle will weigh $2 \times 10^{-12}g$ and there will be 10^{11} particles in a cubic meter of aerosol. If the concentration is homogeneous the spacing between particles will be about $10^{-11/3}$ meter or 214 μm and maximum distances from any given particle will be about half that value. The maximum time required for heat to travel to any parcel of air within the cloud will correspond to a mean square distance equal to the maximum distance away from any particle in the cloud, 107 μm,

$$t=(0.0107cm)^2/((6)(0.187cm^2/s)) = 102 \text{ μsec.}$$

Thermal conduction equilibrium is achieved within several multiples of this time, probably less than one millisecond.

8.   FLARE PARTICLE HEAT TRANSFER

If we want to partition more heat lose into radiation that can reduce contrast and signal to noise transmittance rather than conduction into the air, which has no effect on contrast or signal to noise ratio in atmospheric windows, we can increase particle size and temperature. Increasing particle size will decrease surface area available for both conductive and radiative losses and increase the duration of particle reactions and increase settling velocity. To see how particle size influences particle temperature we first look at the steady state solution for conduction of heat, for example generated by surface and volume reactions in a spherical particle of diameter ξ, into the surrounding air. We assume that the spacing between these large "flare" particles is great enough that heat conduction by any given flare particle is unaffected by heat conduction by surrounding particles. The temperature elevation of the surface of a spherical particle producing $\Delta\dot{H}_v$ watts of power per unit particle volume will be proportional to the particle size squared[7]

$$\Delta T_v = \Delta\dot{H}_v\xi^2 /3 \text{ K}$$

while for surface heating producing $\Delta\dot{H}_s$ watts of power per unit particle surface area the temperature elevation is linearly proportional to particle size[7]

$$\Delta T_s = \Delta\dot{H}_s\xi / K$$

where $K=\kappa\rho \mathbf{c}$ is the conductivity of air. Volume heating could be the result of either exothermic chemical reactions occurring throughout the particle volume or absorption of incident high power density radiation throughout the particle volume, which is often the case when particle size is small compared to the wavelength of the high power radiation. Surface heating could be the result of either exothermic chemical reactions occurring on the particle surface or absorption of incident high power density radiation over the particle surface, which is the case when particle size is large compared to the wavelength of the high power radiation. If we now assume all losses are radiative rather than conductive, energy balance predicts the equilibrium temperature for volume heating will be proportional to $\xi^{1/4}$

$$T_v = \left(\Delta\dot{H}_v\xi/(3 \ \sigma\varepsilon)\right)^{1/4}$$

and the equilibrium temperature for surface heating will be independent of particle size

13

$$T_s = \left(\Delta \dot{H}_s / (\sigma \varepsilon) \right)^{1/4}$$

where ε is the emissivity or absorptivity of the particle. These two expressions for emitted radiation show clearly the particle size dependence of equilibrium temperatures for particles that radiate from their surface rather than their volume. This is true for particles larger than the wavelength of emitted radiation because the emissivity does not depend on particle size. However, when particle size is smaller than the wavelength of emitted radiation, the particles radiate from their volume and a size dependent emissivity could be introduced but it is more appropriate to work with a Rayleigh low frequency expression for particle absorption/emission cross section at the emission wavelengths corresponding to the temperature elevation. We will consider only flare particle sizes greater than the largest emitted wavelengths ~10micrometers and so that emissivity does not depend on particle size.

We turn our attention to the ratio of conductive versus radiative heat losses as a function of particle size, emissivity and temperature. Conductive heat losses are $4\pi K \Delta T \xi$ and radiative heat losses are $4\pi \xi^2 \varepsilon \sigma T^4$ so that the ratio of radiative over conductive heat loses at any specified particle temperature is proportional to particle size. Setting this ratio equal to one we find the particle size ξ_{eq} producing equal conductive and radiative heat losses,

$$\xi_{eq} = K \, \Delta T / \left(\varepsilon \sigma T^4 \right)$$

where $T = T_{ambient} + \Delta T$. At temperature elevations small compared to ambient temperature this particle size increases in proportion to ΔT. For example at an emissivity equal to one and at a $3°K$ temperature elevation above an ambient temperature of $300°K$ the particle size giving equal conductive and radiative heat losses is

$$\xi_{eq}(303°K) = (6.05 \times 10^{-5} cal/cm/s/°K)(4.19j/cal)(3°K)/((5.67 \times 10^{-12} j/s/cm^2/°K^4)(303°K)^4)$$

$$= 0.0159cm.$$

Eventually a maximum size is reached at $\Delta T = T_{ambient}/3$ where

$$\xi_{eq}(max) = (6.05 \times 10^{-5} cal/cm/s/°K)(4.19J/cal)(100°K)/((5.67 \times 10^{-12} J/s/cm^2/°K^4)(400°K)^4)$$

$$= 0.175cm$$

at an ambient temperature of $300°K$. Finally for large temperature elevations this particle size decreases in proportion to $1/T^3$, for example at a temperature elevation of $1500°K$,

$$\xi_{eq}(1800°K) = (6.05 \times 10^{-5} cal/cm/s/°K)(4.19J/cal)(1500°K)/((5.67 \times 10^{-12} J/s/cm^2/°K^4)(1800°K)^4)$$

$$= 0.00639cm$$

By working with larger and hotter flare particles we can increase the portion of heat going into cloud emission rather than heat conduction thereby reducing contrast and signal to noise transmittance. By increasing the $1800°K$ flare particle size in the last expression two orders of magnitude to 0.639cm diameter we can reduce conductive losses to 1% of the radiative losses. But of course such a large sphere will not stay airborne for long and heat convective losses would also have to be considered. Gravitational settling velocities of large particles can be reduced by shaping the particles into tabular forms such as disks or flakes. Particle shapes having the same surface area will lose approximately the

14

same heat and reach the same equilibrium temperature as long as particle heating and emission are surface area dependent and not volume dependent. For example thin disks roughly about 0.9cm in diameter can be expected to perform like the 0.639cm diameter sphere mentioned but with an adjustable gravitational settling velocity and an adjustable surface chemical reaction lifetime depending on the disk thickness chosen.

9.　　　　　　　　FLARE PARTICLE GRAVITATIONAL SETTLING VELICITY

　　　The settling velocity of disk shaped flare particles can be found by looking at their drag coefficients in the laminar, transition and turbulent flow regimes. The gravitational settling velocity is then computed using the equation defining the drag coefficient

$$v = \sqrt{\frac{2mg}{C_D A \rho}}$$

where m is the mass of the flare particle and A is the cross sectional area of the particle transverse to air flow. The disk mass is

$$m = \rho_F \frac{\pi}{4} d^2 t$$

for a disk of density ρ_F, diameter d and thickness t. In the laminar flow regime the drag coefficients are $\frac{64}{\pi \, Re}$ for motion parallel to the disk symmetry axis and $\frac{128d}{3\pi t \, Re}$ for motion along any equatorial axis[9] (perpendicular to the symmetry axis) where the Reynolds number is defined in terms of the major dimension

$$Re = \frac{vd}{\mu}$$

In the turbulent flow regime[10,11] the drag coefficient approaches a value of 1 and the orientation distribution moves from a glide-tumbling motion ultimately to complete random[12] at very large Reynolds numbers. A simple approximation throughout the transition flow regime is a simple combination of these two solutions

$$C_D = \frac{64}{\pi \, Re} + 1 \quad \text{for } \bar{v} \parallel \text{symmetry axis}$$

$$C_D = \frac{128d}{3\pi t Re} + 1 \quad \text{for } \bar{v} \perp \text{symmetry axis.}$$

The settling velocities in the laminar flow regime are

$$v = \frac{\pi t d \rho_F g}{48 \eta} \quad \text{for } \bar{v} \parallel \text{symmetry axis}$$

15

$$v = \frac{\pi t d \rho_F g}{32 \eta} \quad \text{for } \bar{v} \perp \text{symmetry axis}$$

where $\eta = 1.83 \times 10^{-4}$ g/cm/s is the viscosity of air. In the turbulent flow regime the gravitational settling velocities are

$$v = \sqrt{\frac{4 t \rho_F g}{3 \rho}} \quad \text{for } \bar{v} \parallel \text{symmetry axis}$$

$$v = \sqrt{\frac{4 d \rho_F g}{3 \rho}} \quad \text{for } \bar{v} \perp \text{symmetry axis.}$$

Looking at the 0.9 cm diameter flare disk mentioned earlier which puts most of its heat loss into radiation rather than conduction, at a thickness of 20 micrometers and a density of 2 g/cm^3 it would have a glide tumbling settling velocity of approximately

$$v = \sqrt{\frac{4 t \rho_F g}{3 \rho}} = 2 \sqrt{\frac{(20 \times 10^{-4} \text{cm})(2 \text{g/cm}^3)(980 \text{cm/s}^2)}{3(0.0013 \text{g/cm}^3)}}$$

$$= 63.4 \text{ cm/s}$$

corresponding to a Reynolds number

$$\text{Re} = \frac{dv}{\mu} = \frac{(0.9 \text{cm})(63.4 \text{cm/s})}{0.152 \text{cm}^2/\text{s}} = 375$$

which is consistent with the chosen drag coefficient and orientation.

10.   FLARE PARTICLES INCORPORATED INTO AN ATTENUATING SCATTERING AEROSOL

When considering flare particles in smoke, again the four flux theory can be used to relate smoke cloud contrast transmittance and signal to noise transmittance to aerosol extinction coefficient, single scatter albedo, asymmetry factor, concentration path length product and cloud temperature. If we consider the flare particles to be absorbing and therefore emitting while the smoke particles are purely scattering then the temperature of the flare particles can be adjusted independent of the smoke particle temperature and air temperature which can remain at ambient air temperatures if the flare particle size is large enough to lose heat predominantly by radiation rather than conduction into the ambient air. Also it is a simplification to consider the smoke particles as purely scattering because they will not be heated by the radiation given off by the flare particles. The cloud brightness will depend on the concentration of flare particles.

Radiation from the cloud surface is

$$\Delta \dot{H}_{cloud} = \varepsilon_{cloud} S_c \sigma T_F^4$$

16

where S_c is cloud surface area, T_F is flare temperature and $\varepsilon_{cloud} = 1 - R - T_s - T_u$, is cloud emissivity. In the limit of an opaque cloud the emissivity becomes

$$\varepsilon_{cloud} = 1 - R = 1 - \frac{1 - \frac{a}{2}(1+g) - \sqrt{(1-a)(1-ag)}}{\frac{a}{2}(1-g)}$$

This is less than the radiation that would result if flare particles did not radiatively interact by absorbing radiation from other flare particles and self absorption of scattered radiation.

The single scatter albedo of an aerosol cloud containing absorbing/emitting flare particles embedded within a purely scattering aerosol which does not heat up directly due to radiation is found by looking at the fraction of radiation absorbed $(1 - a)$ on average by the mix of flare and scattering particles

$$1 - a = \frac{C_F \alpha_F (1 - a_F) + C_s \alpha_s (1 - a_s)}{C_F \alpha_F + C_s \alpha_s}$$

where C_F, α_F and a_F is the concentration, extinction coefficient, and single scatter albedo of the flare particles and C_s, α_s and a_s are the same for the scattering particles. We note that $a_s = 1$ and because the flare particles are much larger than the scattering smoke particles $C_F \alpha_F \ll C_s \alpha_s$ so that

$$1 - a \approx \frac{C_F \alpha_F (1 - a_F)}{C_s \alpha_s}$$

The absorptivity/emissivity of a black surface is unity permitting a flare particle to emit the greatest power. For such a black particle there will be no scatter cross section component due to reflection or refraction but there will be a scatter component due to diffraction equal to the absorption cross section so we set $a_F = 1/2$ to maximize flare output. For flake or disk shaped flare and scattering particles

$$\alpha \left(m^2/cm^3 \right) = \frac{1}{t(\text{micrometers})}$$

where t is the flare particle thickness t_F or scattering smoke particle thickness t_S in micrometers and the cloud average single scatter albedo becomes

$$a = 1 - \frac{C_F t_s}{2 C_s t_F}$$

The parameter F used in some of the plots is defined as the fraction of radiation absorbed/emitted on average

$$F \equiv 1 - a = \frac{C_F t_s}{2 C_s t_F}$$

and therefore $a = 1 - F$. Because each flare particle absorbs radiation originating from other flare particles as well as self emitted radiation scattered back in its direction, the power required to maintain

17

flare temperature at any given level is not as great as it would be if the flare particles were far apart and acting independently. The surface area of a spherical cloud is πD^2 where D is the cloud diameter and the combined surface area S_F of all flare particles is the cloud volume $\pi D^3/6$ multiplied by the flare particle concentration C_F multiplied by the flare surface area over volume ratio $2\left(\dfrac{\pi d^2}{4}\right)\bigg/\left(\dfrac{\pi d^2}{4}\right)t_F = 2/t_F$

$$S_F = \left(\frac{\pi}{6}D^3\right)\left(\frac{2}{t_F}\right)C_F$$

The value of C_F can be found from the value of F chosen in combination with the flare temperature and cloud optical depth from the plots giving the desired reduction in contrast and signal to noise transmittances.

$$C_F = \frac{2C_s t_F}{t_s}F$$

where the optical depth of the cloud is

$$b = (C_s \alpha_s + C_F \alpha_F)L$$

because $C_s \alpha_s \gg C_F \alpha_F$

$$C_s = \frac{b}{\alpha_s L} = \frac{b t_s}{L}$$

Substituting this value into the equation relating C_F to C_s and F

$$C_F = \frac{2\, b\, t_F}{L}F$$

and the total flare surface area becomes

$$S_F = \left(\frac{\pi}{6}D^3\right)\left(\frac{2}{t_F}\right)\left(\frac{2\, b\, t_F\ F}{L}\right)$$

Letting L = D

$$S_F = \frac{2\pi}{3}D^2\ b\ F$$

and the radiation emitted by the total number of flare particles acting independently is

$$\Delta\dot{H}_{indep} = \varepsilon_{flare}S_F \sigma T_F^{\,4}$$

18

The ratio γ of radiation emitted $\Delta\dot{H}_{cloud}$ by a cloud of high emissivity $\varepsilon_{flare} \approx 1$ flare particles embedded within a scattering cloud divided by the radiation emitted $\Delta\dot{H}_{indep}$ by those flare particles dispersed, so that they do not interact or absorb their own emitted radiation due to scatter, is

$$\gamma = \frac{\varepsilon_{cloud}S_c}{\varepsilon_{flare}S_F} = \frac{(1-R-T_s-T_u)\pi D^2}{(1)\dfrac{2\pi}{3}D^2 \, b \, F}$$

$$\gamma = \frac{3}{2}\frac{(1-R-T_s-T_u)}{b \, F}$$

If a disk shaped cloud had been chosen γ would be less than or equal to one as it should be. Significant reductions below this value would occur only for $F > 0.01$ and $b > 3$. The flare power required per unit volume of flare particles is found by taking the radiation emitted by the cloud and dividing it by the volume of flare particles in the cloud

$$\frac{\Delta\dot{H}_{cloud}}{\left(\dfrac{\pi}{6}D^3\right)C_F} = \frac{(1-R-T_S-T_u)\pi D^2\sigma T_F^4}{\left(\dfrac{\pi}{6}D^3\right)\left(\dfrac{2\,b\,t_F F}{D}\right)} = \frac{3(1-R-T_s-T_u)\sigma T_F^4}{2\,b\,t_F F}$$

which is on the order of 800 W/cm^3 at $T_F = 1000°K$ and $t_F = 0.01$ cm over the parameter range of interest that significantly reduces contrast and signal to noise transmittance; $0.001 \le F \le 0.1$ and $1 \le b \le 3$.

11.   DISCUSSION OF FIGURES

Figures 1-46 address warm clouds where the air temperature is close to the aerosol particle temperature. Contrast transmittance is plotted in Figures 1-10. At several values of single scatter albedo and target over background temperature ratio it appears as a function of smoke optical depth and cloud over background temperature ratio in Figures 1-4. At several values of optical depth and target over background temperature ratio it appears as a function of single scatter albedo and cloud over background temperature ratio in Figures 5-8. All remaining plots assume that target and background temperatures differ by only a few degrees. Contrast transmittance at two cloud over background temperature ratios appears as a function of smoke optical depth and single scatter albedo in Figures 9 and 10. Contrast transmittance ratio (contrast transmittance at the elevated cloud temperature divided by contrast transmittance at ambient cloud temperature) is computed in Figures 11-17. It is computed in Figures 11-13 as a function of cloud over background temperature ratio and optical depth at three single scatter albedos. It is computed in Figures 14 and 15 as a function of cloud over background temperature ratio and single scatter albedo at two optical depths. It is computed in Figures 16 and 17 as a function of optical depth and single scatter albedo at two cloud over background temperature ratios. The ratio of the derivative of contrast transmittance with respect to cloud temperature over the derivative of contrast transmittance with respect to optical depth is shown in Figures 18-24. This ratio is equal to the ratio of the optical depth increment that reduces contrast transmittance by the same amount that a 1 degree Kelvin rise in cloud temperature reduces contrast transmittance. This optical depth increment is plotted as a function of optical depth and single scatter albedo in Figures 18 and 19, as a function of cloud temperature and single scatter albedo in Figures 20 and 21 and as a function of cloud temperature and optical depth in Figures 22-24. Signal to noise transmittance is shown in Figures 25-31. It is computed as a function of optical depth and cloud over background temperature ratio in Figures 25-27, as a function

of single scatter albedo and cloud over background temperature ratio in Figures 28 and 29 and as a function of optical depth and single scatter albedo in Figures 30 and 31. Signal to noise transmittance ratio (signal to noise transmittance at the elevated cloud temperature divided by signal to noise transmittance at ambient cloud temperature) is computed in Figures 32-38. It is computed in Figures 32-34 as a function of cloud over background temperature ratio and optical depth at three single scatter albedos. It is computed in Figures 35 and 36 as a function of cloud over background temperature ratio and single scatter albedo at two optical depths. It is computed in Figures 37 and 38 as a function of optical depth and single scatter albedo at two cloud over background temperature ratios. The ratio of the derivative of signal to noise transmittance with respect to cloud temperature over the derivative of signal to noise transmittance with respect to optical depth is shown in Figures 39-46. This ratio is equal to the ratio of the optical depth increment that reduces signal to noise transmittance by the same amount that a 1 degree Kelvin rise in cloud temperature reduces signal to noise transmittance. This optical depth increment is plotted as a function of optical depth and single scatter albedo in Figures 39 and 40, as a function of cloud temperature and single scatter albedo in Figures 41-43 and as a function of cloud temperature and optical depth in Figures 44-46.

Figures 47-70 address flare particles dispersed within a scattering smoke that is not heated by the flare particles. Figures 47-62 show contrast transmittance and signal to noise transmittance as a function of two of the following independent variables while holding the remaining variable fixed ; optical depth, logarithm base 10 of the ratio of flare over background temperature and logarithm base 10 of F or equivalently rf/R which is equal one minus the single scatter albedo. Figures 63-66 show the ratio of the derivative of signal to noise transmittance with respect to the ratio of flare over background temperature divided by the derivative of signal to noise transmittance with respect to optical depth. This is equal to the optical depth increment required to reduce signal to noise transmittance by the same amount that a unit increment in the ratio of flare over background temperature reduces signal to noise transmittance. Figures 67-70 show the ratio of the derivative of signal to noise transmittance with respect to F divided by the derivative of signal to noise transmittance with respect to optical depth. This is equal to the optical depth increment required to reduce signal to noise transmittance by the same amount that a unit increment in F reduces signal to noise transmittance.

12.   CONCLUSIONS

Smoke optical depth can be decreased modestly, on the order of 0.005 per degree Kelvin, by increasing cloud temperature and emission. This number is based on reductions in contrast transmittance and signal to noise transmittance over a wide range of optical depths and single scatter albedos. However the energy required to raise the temperature of the cloud is considerable, about 10,000 joules per gram of aerosol per degree Kelvin at a concentration of 0.13g/m^3, because of the huge ratio of air mass (air density is 1300g/m^3) to aerosol mass. In addition cloud buoyancy quickly moves the cloud up out of the line of sight. A ten meter diameter cloud that is $3°\text{K}$ warmer than the surrounding air will rise at a velocity of 3.6m/s. Such velocities of rise are in the turbulent flow regime and will cause the cloud to break up even if it is rising through relatively still air.

An alternative method to increase cloud emission has been proposed here that is much more effective. Cloud emission can be significantly increased by incorporating large tabular (flake or disk) shaped flare particles into a scattering smoke because a much larger fraction of heat loss is put into radiation rather than conduction into air that does not emit in atmospheric windows. Flare particle size, shape, temperature, concentration and the ratio of flare particles to scattering particles have been optimized to minimize conductive heat loss and settling velocity while minimizing smoke contrast transmittance and signal to noise transmittance. Optical depth reductions on the order of 0.3 for a $300°\text{K}$ increase in flare particle temperature are possible over a wide range of optical depths at flare temperatures

20

greater than 900°K and F values greater than 0.001. We define F to be half the ratio of flare particle concentration extinction coefficient product divided by scattering particle concentration extinction coefficient product. Optical depth reductions on the order of 0.05 for a 0.001 increase in F have been computed over a wide range of optical depths at flare temperatures above 900°K and F values greater than 0.001. Above flare particle temperatures of 400°K the ratio of radiation lose over conduction lose increases with increasing particle size and temperature. For example a 0.9cm diameter flare disk at a temperature of 1800°K will lose 99% of its heat by radiation and have a settling velocity of 63.4cm/s at a thickness of 20 micrometers and a density of 2g/cc. To further reduce settling velocity flare thickness and density might be reduced since settling velocity is proportional to the square root of the product of thickness and density. It is also possible to create flare particle shapes that spin as they fall to reduce settling velocity. Power production requirements for flare particles of a given thickness are obtained by looking at the appropriate figure and then choosing a combination of optical depth, F and flare temperature that reduces contrast transmittance or signal to noise transmittance to the desired level. For example flare particles produce 800w/cc at a temperature of 1000°K and thickness of 100 micrometers over the range of optical depths and F values of interest.

In closing, certain limitations should be mentioned. This study has represented emitted power as proportional to temperature raised to the fourth power without addressing wavelength dependence which manifests itself as peak power output moving progressively to shorter wavelengths as flare temperature is increased. Also all flare particle calculations of contrast transmittance and signal to noise transmittance have assumed a scattering nonabsorbing smoke. Real smoke particles also absorb and reemit radiation in at least portions of the infrared but this effect could not be studied using the simplified radiative transfer equations presented here. Finally, clouds were assumed to be homogeneous walls or blankets while in face the clouds are inhomogeneous and finite in all dimensions. This means that effects due to flux transported through the sides of clouds were not investigated nor were the effects of clutter due to aerosol concentration inhomogeneities studied.
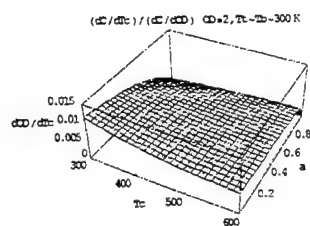
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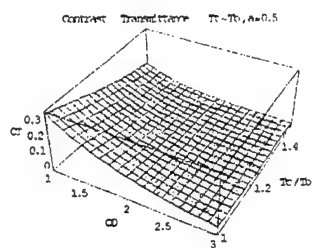
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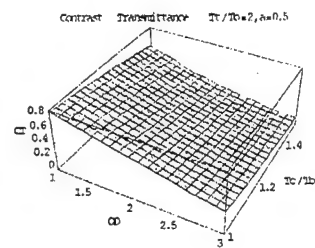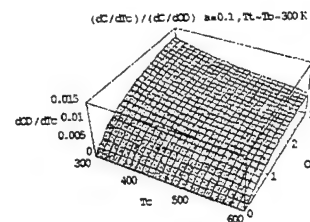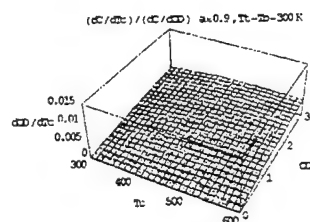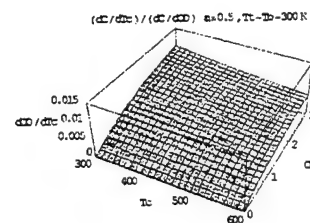

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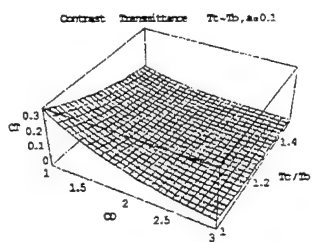
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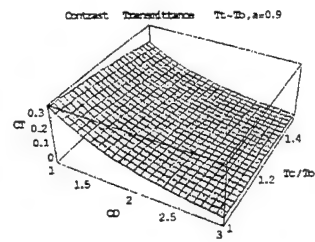
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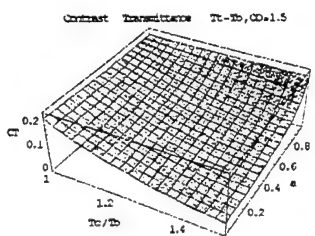
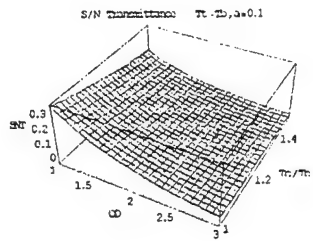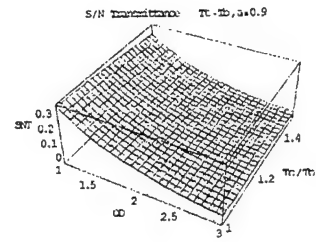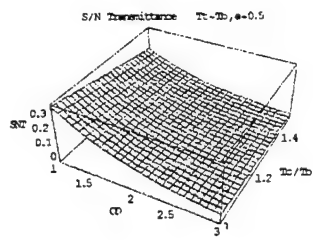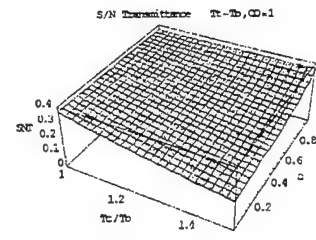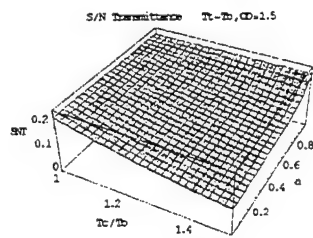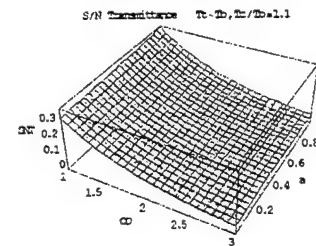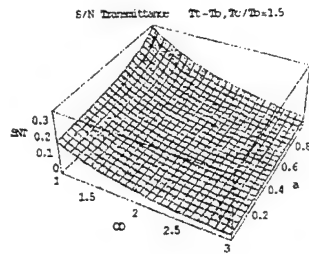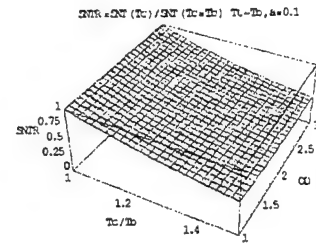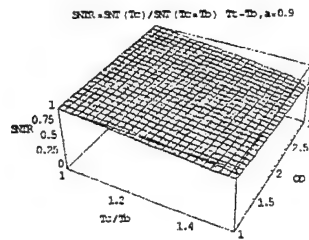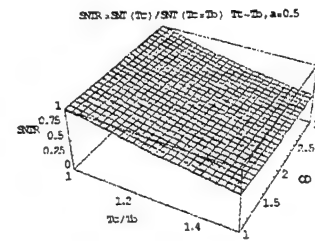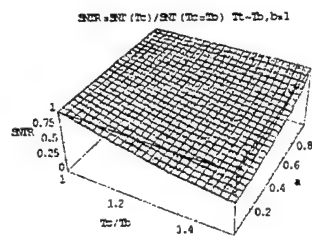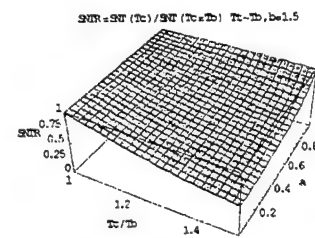

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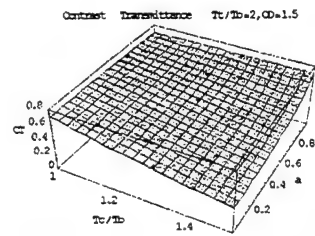
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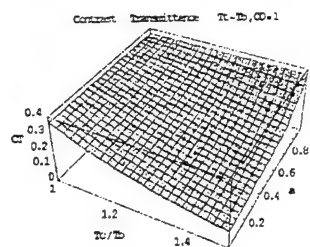
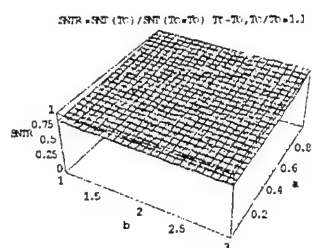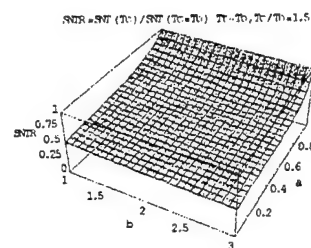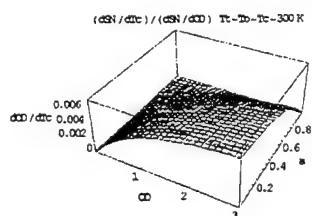
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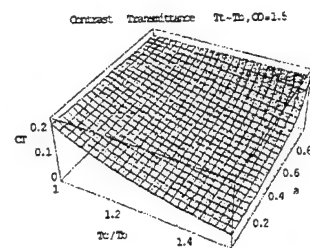
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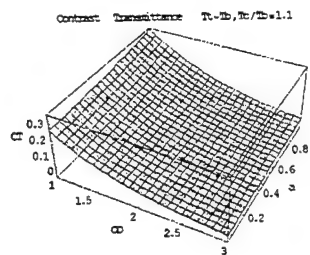
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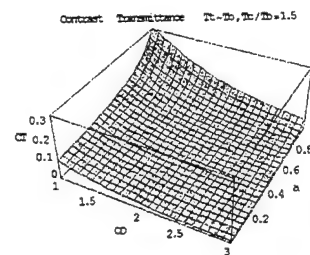
Figure 11

Figure 12

Figure 13

$CR=CT(Tc)/CT(Tc=Tb)$ $Tt=Tb, a=0.5$

Figure 14

$CR=CT(Tc)/CT(Tc=Tb)$ $Tt=Tb, b=1$

Figure 15

$CR=CT(Tc)/CT(Tc=Tb)$ $Tt=Tb, b=1.5$

Figure 16

$CR=CT(Tc)/CT(Tc=Tb)$ $Tt=Tb, Tc/Tb=1.1$

Figure 17

$CR=CT(Tc)/CT(Tc=Tb)$ $Tt=Tb, Tc/Tb=1.5$

Figure 18

$(dC/dTc)/(dC/dD)$ $Tt=Tb=Tc=300\,K$

Figure 19

$(dC/dTc)/(dC/dOD)$ Tt~Tb~300, Tc~400

Figure 20

$(dC/dTc)/(dC/dOD)$ OD=1, Tt~Tb~300 K

Figure 21

$(dC/dTc)/(dC/dOD)$ OD=2, Tt~Tb~300 K

Figure 22

$(dC/dTc)/(dC/dOD)$ a=0.1, Tt~Tb~300 K

Figure 23

$(dC/dTc)/(dC/dOD)$ a=0.9, Tt~Tb~300 K

Figure 24

$(dC/dTc)/(dC/dOD)$ a=0.5, Tt~Tb~300 K

Figure 25

S/N Transmittance Tt - Tb, a=0.1

Figure 26

S/N Transmittance Tt - Tb, a=0.9

Figure 27

S/N Transmittance Tt - Tb, a=0.5

Figure 28

S/N Transmittance Tt - Tb, CD=1

Figure 29

S/N Transmittance Tt - Tb, CD=1.5

Figure 30

S/N Transmittance Tt - Tb, Tc/Tb=1.1

26

Figure 31

S/N Transmittance   Tt~Tb , Tc/Tb=1.5

Figure 32

SNR = SNT(Tc)/SNT(Tc=Tb) Tt~Tb , a=0.1

Figure 33

SNR = SNT(Tc)/SNT(Tc=Tb) Tt~Tb , a=0.9

Figure 34

SNR = SNT(Tc)/SNT(Tc=Tb) Tt~Tb , a=0.5

Figure 35

SNR = SNT(Tc)/SNT(Tc=Tb) Tt~Tb , b=1

Figure 36

SNR = SNT(Tc)/SNT(Tc=Tb) Tt~Tb , b=1.5

27

Figure 37

SNTR = SNT (Tc) / SNT (Tc = Tb)   Tt = Tb, Tc / Tb = 1.1

Figure 38

SNTR = SNT (Tc) / SNT (Tc = Tb)   Tt = Tb, Tc / Tb = 1.5

Figure 39

(dSN / dTc) / (dSN / dCO) Tt = Tb = Tc = 300 K

Figure 40

(dSN / dTc) / (dSN / dCO) Tt = Tb = 300, Tc = 600

Figure 41

(dSN / dTc) / (dSN / dCO) CO = 1, Tt = Tb = 300 K

Figure 42

(dSN / dTc) / (dSN / dCO) CO = 3, Tt = Tb = 300 K

28

Figure 43

(dSN/dTc)/(dSN/dOD) OD≈2, Tt≈Tb≈300 K

Figure 44

(dSN/dTc)/(dSN/dOD) a≈0.1, Tt≈Tb≈300 K

Figure 45

(dSN/dTc)/(dSN/dOD) a≈0.9, Tt≈Tb≈300 K

Figure 46

(dSN/dTc)/(dSN/dOD) a≈0.5, Tt≈Tb≈300 K

Figure 47

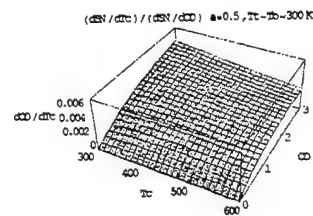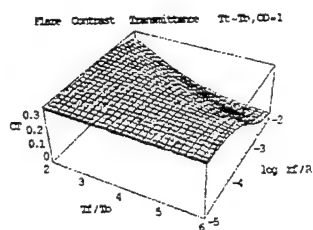Flare Contrast Transmittance Tt≈Tb, OD=1
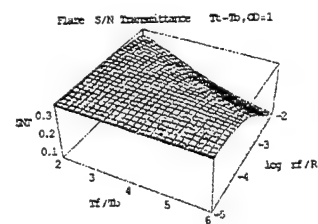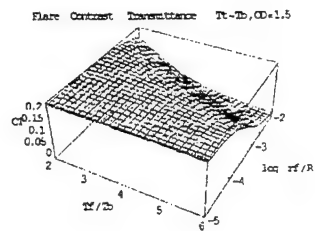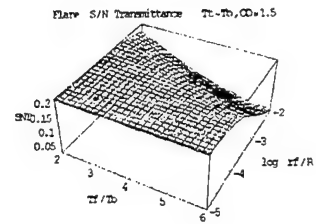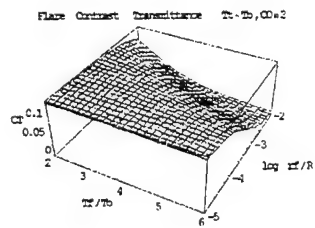
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Flare S/N Transmittance Tt≈Tb, OD=1

Figure 49

Figure 50

Figure 51

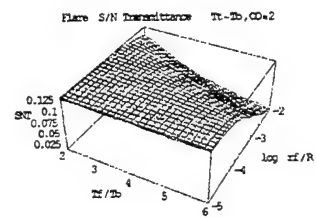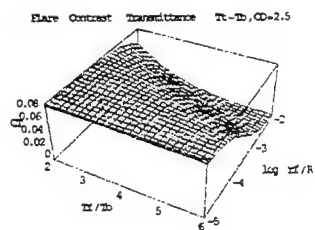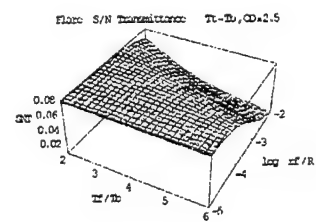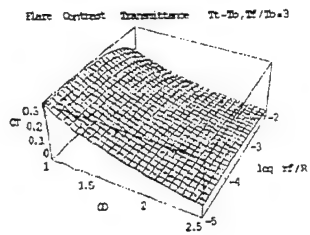

Figure 52

Figure 53

Figure 54

Figure 55

Flare Contrast Transmittance   Tt~Tb,Tf/Tb=3

Figure 56

Flare S/N Transmittance   Tt~Tb,Tf/Tb=3

Figure 57

Flare Contrast Transmittance   Tt~Tb,Tf/Tb=6

Figure 58

Flare S/N Transmittance   Tt~Tb,Tf/Tb=6

Figure 59

Flare Contrast Transmittance   Tt~Tb,rf/R=.01

Figure 60

Flare S/N Transmittance   Tt~Tb,rf/R=.01

31

Figure 61

Flare Contrast Transmittance   Tt-Tb, rf/R=.001

Figure 62

Flare S/N Transmittance   Tt-Tb, rf/R=.001

Figure 63

Flare (dSN/dTfb)/(dSN/dTD) Tt-Tb,Tf/Tb=3

Figure 64

Flare (dSN/dTfb)/(dSN/dTD) Tt-Tb,Tf/Tb=4

Figure 65

Flare (dSN/dTfb)/(dSN/dTD) Tt-Tb,Tf/Tb=5

Figure 66

Flare (dSN/dTfb)/(dSN/dTD) Tt-Tb,Tf/Tb=6

32

Figure 67

Flare (dBN/dF)/(dBN/dBD) Tt~Tb,Tf/Tb=2

Figure 68

Flare (dBN/dF)/(dBN/dBD) Tt~Tb,Tf/Tb=4

Figure 69

Flare (dBN/dF)/(dBN/dBD) Tt~Tb,Tf/Tb=5

Figure 70

Flare (dBN/dF)/(dBN/dBD) Tt~Tb,Tf/Tb=6

33

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SOURCE CODE

Cloud Infrared Emission versus Attenuation Codes

```
Program Darpacr1
g = 0.75;
temptargovrbkg = 1.01;
p = a(1 + g)/2;
q = a(1 - g)/2;
z = Sqrt[(1 - a)(1 - a g)];
Tu = Exp[-b];
T = 2z/((1 - p + z)Exp[z b] - (1 - p - z)Exp[-z b]);
R = q(Exp[z b] - Exp[-z b])/((1 - p + z)Exp[z b] - (1 - p - z)Exp[-z b]);
(*contrast ratio for cloud at tempcloudovrbkg divided by contrast for cloud \
of same properties at background temperature*)
CR = (Tu(1 + temptargovrbkg^4) + 2(R + T - Tu) +
      2(1 - R - T))/(Tu(1 + temptargovrbkg^4) + 2(R + T - Tu) +
      2(1 - R - T)tempcloudovrbkg^4 );
a = 0.5;
Plot3D[CR, {tempcloudovrbkg, 1, 1.5}, {b, 1, 3},
  PlotLabel -> "CR=CT(Tc)/CT(Tc=Tb) Tt~Tb,a=0.5",
  AxesLabel -> {"Tc/Tb", "OD", "CR "}, PlotPoints -> 25, PlotRange -> {0, 1}];
a = 0.1;
Plot3D[CR, {tempcloudovrbkg, 1, 1.5}, {b, 1, 3},
  PlotLabel -> "CR=CT(Tc)/CT(Tc=Tb) Tt~Tb,a=0.1",
  AxesLabel -> {"Tc/Tb", "OD", "CR "}, PlotPoints -> 25, PlotRange -> {0, 1}];
a = 0.9;
Plot3D[CR, {tempcloudovrbkg, 1, 1.5}, {b, 1, 3},
  PlotLabel -> "CR=CT(Tc)/CT(Tc=Tb) Tt~Tb,a=0.9",
  AxesLabel -> {"Tc/Tb", "OD", "CR "}, PlotPoints -> 25, PlotRange -> {0, 1}]
```

```
Program Darpacr11
g = 0.75;
temptargovrbkg = 1.01;
p = a(1 + g)/2;
q = a(1 - g)/2;
z = Sqrt[(1 - a)(1 - a g)];
Tu = Exp[-b];
T = 2z/((1 - p + z)Exp[z b] - (1 - p - z)Exp[-z b]);
R = q(Exp[z b] - Exp[-z b])/((1 - p + z)Exp[z b] - (1 - p - z)Exp[-z b]);
(*contrast ratio for cloud at tempcloudovrbkg divided by contrast for cloud \
of same properties at background temperature*)
CR = (Tu(1 + temptargovrbkg^4) + 2(R + T - Tu) +
      2(1 - R - T))/(Tu(1 + temptargovrbkg^4) + 2(R + T - Tu) +
      2(1 - R - T)tempcloudovrbkg^4 );
SNTR = CR^.5;
a = 0.5;
Plot3D[SNTR, {tempcloudovrbkg, 1, 1.5}, {b, 1, 3},
  PlotLabel -> "SNTR=SNT(Tc)/SNT(Tc=Tb) Tt~Tb,a=0.5",
  AxesLabel -> {"Tc/Tb", "OD", "SNTR "}, PlotPoints -> 25,
  PlotRange -> {0, 1}];
a = 0.1;
Plot3D[SNTR, {tempcloudovrbkg, 1, 1.5}, {b, 1, 3},
  PlotLabel -> "SNTR=SNT(Tc)/SNT(Tc=Tb) Tt~Tb,a=0.1",
  AxesLabel -> {"Tc/Tb", "OD", "SNTR "}, PlotPoints -> 25,
  PlotRange -> {0, 1}];
a = 0.9;
Plot3D[SNTR, {tempcloudovrbkg, 1, 1.5}, {b, 1, 3},
  PlotLabel -> "SNTR=SNT(Tc)/SNT(Tc=Tb) Tt~Tb,a=0.9",
  AxesLabel -> {"Tc/Tb", "OD", "SNTR "}, PlotPoints -> 25,
  PlotRange -> {0, 1}]
```

```
Program Darpacr2
g = 0.75;
temptargovrbkg = 1.01;
p = a(1 + g)/2;
q = a(1 - g)/2;
z = Sqrt[(1 - a)(1 - a g)];
Tu = Exp[-b];
T = 2z/((1 - p + z)Exp[z b] - (1 - p - z)Exp[-z b]);
R = q(Exp[z b] - Exp[-z b])/((1 - p + z)Exp[z b] - (1 - p - z)Exp[-z b]);
(*contrast ratio for cloud at tempcloudovrbkg divided by contrast for cloud \
of same properties at background temperature*)
CR = (Tu(1 + temptargovrbkg^4) + 2(R + T - Tu) +
      2(1 - R - T))/(Tu(1 + temptargovrbkg^4) + 2(R + T - Tu) +
      2(1 - R - T)tempcloudovrbkg^4 );
b = 1;
Plot3D[CR, {tempcloudovrbkg, 1, 1.5}, {a, 0.0001, .9999},
  PlotLabel -> "CR=CT(Tc)/CT(Tc=Tb) Tt~Tb,b=1",
  AxesLabel -> {"Tc/Tb", "a", "CR "}, PlotPoints -> 25, PlotRange -> {0, 1}];
b = 1.5;
Plot3D[CR, {tempcloudovrbkg, 1, 1.5}, {a, 0.0001, .9999},
  PlotLabel -> "CR=CT(Tc)/CT(Tc=Tb) Tt~Tb,b=1.5",
  AxesLabel -> {"Tc/Tb", "a", "CR "}, PlotPoints -> 25, PlotRange -> {0, 1}]
```

```
Program Darpacr22
g = 0.75;
temptargovrbkg = 1.01;
p = a(1 + g)/2;
q = a(1 - g)/2;
z = Sqrt[(1 - a)(1 - a g)];
Tu = Exp[-b];
T = 2z/((1 - p + z)Exp[z b] - (1 - p - z)Exp[-z b]);
R = q(Exp[z b] - Exp[-z b])/((1 - p + z)Exp[z b] - (1 - p - z)Exp[-z b]);
(*contrast ratio for cloud at tempcloudovrbkg divided by contrast for cloud \
of same properties at background temperature*)
CR = (Tu(1 + temptargovrbkg^4) + 2(R + T - Tu) +
      2(1 - R - T))/(Tu(1 + temptargovrbkg^4) + 2(R + T - Tu) +
      2(1 - R - T)tempcloudovrbkg^4 );
SNTR = CR^.5;
b = 1;
Plot3D[SNTR, {tempcloudovrbkg, 1, 1.5}, {a, .0001, .9999},
  PlotLabel -> "SNTR=SNT(Tc)/SNT(Tc=Tb) Tt~Tb,b=1",
  AxesLabel -> {"Tc/Tb", "a", "SNTR "}, PlotPoints -> 25, PlotRange -> {0, 1}];
b = 1.5;
Plot3D[SNTR, {tempcloudovrbkg, 1, 1.5}, {a, .0001, .9999},
  PlotLabel -> "SNTR=SNT(Tc)/SNT(Tc=Tb) Tt~Tb,b=1.5",
  AxesLabel -> {"Tc/Tb", "a", "SNTR "}, PlotPoints -> 25, PlotRange -> {0, 1}]
```

```
Program Darpacr3
g = 0.75;
temptargovrbkg = 1.01;
p = a(1 + g)/2;
q = a(1 - g)/2;
z = Sqrt[(1 - a)(1 - a g)];
Tu = Exp[-b];
T = 2z/((1 - p + z)Exp[z b] - (1 - p - z)Exp[-z b]);
R = q(Exp[z b] - Exp[-z b])/((1 - p + z)Exp[z b] - (1 - p - z)Exp[-z b]);
(*contrast ratio for cloud at tempcloudovrbkg divided by contrast for cloud \
of same properties at background temperature*)
CR = (Tu(1 + temptargovrbkg^4) + 2(R + T - Tu) +
        2(1 - R - T))/(Tu(1 + temptargovrbkg^4) + 2(R + T - Tu) +
        2(1 - R - T)tempcloudovrbkg^4 );
tempcloudovrbkg = 1.1;
Plot3D[CR, {b, 1, 3}, {a, 0.0001, .9999},
  PlotLabel -> "CR=CT(Tc)/CT(Tc=Tb) Tt~Tb,Tc/Tb=1.1",
  AxesLabel -> {"b", "a", "CR "}, PlotPoints -> 25, PlotRange -> {0, 1}];
tempcloudovrbkg = 1.5;
Plot3D[CR, {b, 1, 3}, {a, 0.0001, .9999},
  PlotLabel -> "CR=CT(Tc)/CT(Tc=Tb) Tt~Tb,Tc/Tb=1.5",
  AxesLabel -> {"b", "a", "CR "}, PlotPoints -> 25, PlotRange -> {0, 1}]
```

```
Program Darpacr33
g = 0.75;
temptargovrbkg = 1.01;
p = a(1 + g)/2;
q = a(1 - g)/2;
z = Sqrt[(1 - a)(1 - a g)];
Tu = Exp[-b];
T = 2z/((1 - p + z)Exp[z b] - (1 - p - z)Exp[-z b]);
R = q(Exp[z b] - Exp[-z b])/((1 - p + z)Exp[z b] - (1 - p - z)Exp[-z b]);
(*contrast ratio for cloud at tempcloudovrbkg divided by contrast for cloud \
of same properties at background temperature*)
CR = (Tu(1 + temptargovrbkg^4) + 2(R + T - Tu) +
      2(1 - R - T))/(Tu(1 + temptargovrbkg^4) + 2(R + T - Tu) +
      2(1 - R - T)tempcloudovrbkg^4 );
SNTR = CR^.5;
tempcloudovrbkg = 1.1;
Plot3D[SNTR, {b, 1, 3}, {a, .0001, .9999},
  PlotLabel -> "SNTR=SNT(Tc)/SNT(Tc=Tb) Tt~Tb,Tc/Tb=1.1",
  AxesLabel -> {"b", "a", "SNTR "}, PlotPoints -> 25, PlotRange -> {0, 1}];
tempcloudovrbkg = 1.5;
Plot3D[SNTR, {b, 1, 3}, {a, .0001, .9999},
  PlotLabel -> "SNTR=SNT(Tc)/SNT(Tc=Tb) Tt~Tb,Tc/Tb=1.5",
  AxesLabel -> {"b", "a", "SNTR "}, PlotPoints -> 25, PlotRange -> {0, 1}]
```

```
Program Darpact1
g = 0.75;
p = a(1 + g)/2;
q = a(1 - g)/2;
z = Sqrt[(1 - a)(1 - a g)];
Tu = Exp[-b];
T = 2z/((1 - p + z)Exp[z b] - (1 - p - z)Exp[-z b]);
R = q(Exp[z b] - Exp[-z b])/((1 - p + z)Exp[z b] - (1 - p - z)Exp[-z b]);
(*Note that contrast transmittance is just contrast ratio -
   contrast with cloud/without cloud*)
CT = Tu(temptargovrbkg^4 + 1)/(Tu(temptargovrbkg^4 + 1) + 2(R + T - Tu) +
        2(1 - R - T)tempcloudovrbkg^4 );
a = 0.5;
temptargovrbkg = 1.0001;
g1 = Plot3D[CT, {b, 1, 3}, {tempcloudovrbkg, 1, 1.5},
   PlotLabel -> "Contrast Transmittance Tt~Tb,a=0.5",
   AxesLabel -> {"OD", "Tc/Tb", "CT "}, PlotPoints -> 25];
a = 0.1;
temptargovrbkg = 1.0001;
g1 = Plot3D[CT, {b, 1, 3}, {tempcloudovrbkg, 1, 1.5},
   PlotLabel -> "Contrast Transmittance Tt~Tb,a=0.1",
   AxesLabel -> {"OD", "Tc/Tb", "CT "}, PlotPoints -> 25];
a = 0.9;
temptargovrbkg = 1.0001;
g1 = Plot3D[CT, {b, 1, 3}, {tempcloudovrbkg, 1, 1.5},
   PlotLabel -> "Contrast Transmittance Tt~Tb,a=0.9",
   AxesLabel -> {"OD", "Tc/Tb", "CT "}, PlotPoints -> 25]
```

```
Program Darpact11
g = 0.75;
p = a(1 + g)/2;
q = a(1 - g)/2;
z = Sqrt[(1 - a)(1 - a g)];
Tu = Exp[-b];
T = 2z/((1 - p + z)Exp[z b] - (1 - p - z)Exp[-z b]);
R = q(Exp[z b] - Exp[-z b])/((1 - p + z)Exp[z b] - (1 - p - z)Exp[-z b]);
(*Note that contrast transmittance is just contrast ratio -
   contrast with cloud/without cloud*)
CT = Tu(temptargovrbkg^4 + 1)/(Tu(temptargovrbkg^4 + 1) + 2(R + T - Tu) +
        2(1 - R - T)tempcloudovrbkg^4 );
SNT = (Tu CT)^.5;
a = 0.5;
temptargovrbkg = 1.0001;
g1 = Plot3D[SNT, {b, 1, 3}, {tempcloudovrbkg, 1, 1.5},
   PlotLabel -> "S/N Transmittance Tt~Tb,a=0.5",
   AxesLabel -> {"OD", "Tc/Tb", "SNT "}, PlotPoints -> 25];
a = 0.1;
temptargovrbkg = 1.0001;
g1 = Plot3D[SNT, {b, 1, 3}, {tempcloudovrbkg, 1, 1.5},
   PlotLabel -> "S/N Transmittance Tt~Tb,a=0.1",
   AxesLabel -> {"OD", "Tc/Tb", "SNT "}, PlotPoints -> 25];
a = 0.9;
temptargovrbkg = 1.0001;
g1 = Plot3D[SNT, {b, 1, 3}, {tempcloudovrbkg, 1, 1.5},
   PlotLabel -> "S/N Transmittance Tt~Tb,a=0.9",
   AxesLabel -> {"OD", "Tc/Tb", "SNT "}, PlotPoints -> 25]
```

```
Program Darpact2
g = 0.75;
p = a(1 + g)/2;
q = a(1 - g)/2;
z = Sqrt[(1 - a)(1 - a g)];
Tu = Exp[-b];
T = 2z/((1 - p + z)Exp[z b] - (1 - p - z)Exp[-z b]);
R = q(Exp[z b] - Exp[-z b])/((1 - p + z)Exp[z b] - (1 - p - z)Exp[-z b]);
(*Note that contrast transmittance is just contrast ratio -
    contrast with cloud/without cloud*)
CT = Tu(temptargovrbkg^4 + 1)/(Tu(temptargovrbkg^4 + 1) + 2(R + T - Tu) +
        2(1 - R - T)tempcloudovrbkg^4 );
b = 1;
temptargovrbkg = 1.0001;
Plot3D[CT, {tempcloudovrbkg, 1, 1.5}, {a, .999, 0.0001},
  PlotLabel -> "Contrast Transmittance Tt~Tb,OD=1",
  AxesLabel -> {"Tc/Tb", "a", "CT "}, PlotPoints -> 25];
Show[%, PlotRange -> {0, .4}];
b = 1.5;
temptargovrbkg = 1.0001;
Plot3D[CT, {tempcloudovrbkg, 1, 1.5}, {a, .999, 0.0001},
  PlotLabel -> "Contrast Transmittance Tt~Tb,OD=1.5",
  AxesLabel -> {"Tc/Tb", "a", "CT "}, PlotPoints -> 25];
Show[%, PlotRange -> {0, .25}]
```

```
Program Darpact22
g = 0.75;
p = a(1 + g)/2;
q = a(1 - g)/2;
z = Sqrt[(1 - a)(1 - a g)];
Tu = Exp[-b];
T = 2z/((1 - p + z)Exp[z b] - (1 - p - z)Exp[-z b]);
R = q(Exp[z b] - Exp[-z b])/((1 - p + z)Exp[z b] - (1 - p - z)Exp[-z b]);
(*Note that contrast transmittance is just contrast ratio -
    contrast with cloud/without cloud*)
CT = Tu(temptargovrbkg^4 + 1)/(Tu(temptargovrbkg^4 + 1) + 2(R + T - Tu) +
        2(1 - R - T)tempcloudovrbkg^4 );
SNT = (Tu CT)^.5;
b = 1;
temptargovrbkg = 1.0001;
g1 = Plot3D[SNT, {tempcloudovrbkg, 1, 1.5}, {a, .9999, 0.00001},
    PlotLabel -> "S/N Transmittance Tt~Tb,OD=1",
    AxesLabel -> {"Tc/Tb", "a", "SNT "}, PlotPoints -> 25];
Show[%, PlotRange -> {0, .4}];
b = 1.5;
g1 = Plot3D[SNT, {tempcloudovrbkg, 1, 1.5}, {a, .9999, 0.00001},
    PlotLabel -> "S/N Transmittance Tt~Tb,OD=1.5",
    AxesLabel -> {"Tc/Tb", "a", "SNT "}, PlotRange -> {0, .25},
    PlotPoints -> 25]
```

```
Program Darpact3
g = 0.75;
p = a(1 + g)/2;
q = a(1 - g)/2;
z = Sqrt[(1 - a)(1 - a g)];
Tu = Exp[-b];
T = 2z/((1 - p + z)Exp[z b] - (1 - p - z)Exp[-z b]);
R = q(Exp[z b] - Exp[-z b])/((1 - p + z)Exp[z b] - (1 - p - z)Exp[-z b]);
(*Note that contrast transmittance is just contrast ratio -
   contrast with cloud/without cloud*)
CT = Tu(temptargovrbkg^4 + 1)/(Tu(temptargovrbkg^4 + 1) + 2(R + T - Tu) +
        2(1 - R - T)tempcloudovrbkg^4 );
tempcloudovrbkg = 1.1;
temptargovrbkg = 1.0001;
g1 = Plot3D[CT, {b, 1, 3}, {a, 0.00001, .99999},
   PlotLabel -> "Contrast Transmittance Tt~Tb,Tc/Tb=1.1",
   AxesLabel -> {"OD", "a", "CT "}, PlotPoints -> 25];
tempcloudovrbkg = 1.5;
g1 = Plot3D[CT, {b, 1, 3}, {a, 0.00001, .99999},
   PlotLabel -> "Contrast Transmittance Tt~Tb,Tc/Tb=1.5",
   AxesLabel -> {"OD", "a", "CT "}, PlotPoints -> 25, PlotRange -> {0, .35}]
```

```
Program Darpact33
g = 0.75;
p = a(1 + g)/2;
q = a(1 - g)/2;
z = Sqrt[(1 - a)(1 - a g)];
Tu = Exp[-b];
T = 2z/((1 - p + z)Exp[z b] - (1 - p - z)Exp[-z b]);
R = q(Exp[z b] - Exp[-z b])/((1 - p + z)Exp[z b] - (1 - p - z)Exp[-z b]);
(*Note that contrast transmittance is just contrast ratio -
   contrast with cloud/without cloud*)
CT = Tu(temptargovrbkg^4 + 1)/(Tu(temptargovrbkg^4 + 1) + 2(R + T - Tu) +
        2(1 - R - T)tempcloudovrbkg^4 );
SNT = (Tu CT)^.5;
tempcloudovrbkg = 1.1;
temptargovrbkg = 1.0001;
g1 = Plot3D[SNT, {b, 1, 3}, {a, .9999, 0.00001},
   PlotLabel -> "S/N Transmittance Tt~Tb,Tc/Tb=1.1",
   AxesLabel -> {"OD", "a", "SNT "}, PlotPoints -> 25];
tempcloudovrbkg = 1.5;
g1 = Plot3D[SNT, {b, 1, 3}, {a, .9999, 0.00001},
   PlotLabel -> "S/N Transmittance Tt~Tb,Tc/Tb=1.5",
   AxesLabel -> {"OD", "a", "SNT "}, PlotPoints -> 25]
```

```
Program Darpaemit1
(*found plots are independent of g as expected for homogeneous illumination*)
g = 0.75;
tback = 300;
ttarg = 301;
p = a(1 + g)/2;
q = a(1 - g)/2;
z = Sqrt[(1 - a)(1 - a g)];
Tu = Exp[-b];
T = 2z/((1 - p + z)Exp[z b] - (1 - p - z)Exp[-z b]);
R = q(Exp[z b] - Exp[-z b])/((1 - p + z)Exp[z b] - (1 - p - z)Exp[-z b]);
CC = Tu(ttarg^4 - tback^4)/(Tu(ttarg^4 + tback^4) + 2 tback^4(R + T - Tu) +
        2(1 - R - T)tcloud^4 );
dd = D[CC, tcloud]/D[CC, b];
tcloud = 300;
(*FindMinimum[(CC - .02)^2, {b, 1}]*)
Plot3D[dd, {b, 0.0001, 3}, {a, 0.0001, 0.9999},
  PlotLabel -> "(dC/dTc)/(dC/dOD) Tt~Tb~Tc~300K",
  AxesLabel -> {"OD", " a", "dOD/dTc "}, PlotPoints -> 25,
  PlotRange -> {0, .015}];
tcloud = 400;
Plot3D[dd, {b, 0.0001, 3}, {a, 0.0001, 0.9999},
  PlotLabel -> "(dC/dTc)/(dC/dOD) Tt~Tb~300,Tc~400",
  AxesLabel -> {"OD", " a", "dOD/dTc "}, PlotPoints -> 25,
  PlotRange -> {0, .015}]
```

```
Program Darpaemit11
(*found plots are independent of g as expected for homogeneous illumination*)
g = 0.75;
tback = 300;
ttarg = 301;
p = a(1 + g)/2;
q = a(1 - g)/2;
z = Sqrt[(1 - a)(1 - a g)];
Tu = Exp[-b];
T = 2z/((1 - p + z)Exp[z b] - (1 - p - z)Exp[-z b]);
R = q(Exp[z b] - Exp[-z b])/((1 - p + z)Exp[z b] - (1 - p - z)Exp[-z b]);
CC = Tu(ttarg^4 -
        tback^4)/(Tu(ttarg^4 + tback^4) + 2 tback^4(R + T - Tu) +
        2(1 - R - T)tcloud^4 )^.5;
dd = D[CC, tcloud]/D[CC, b];
tcloud = 300;
(*FindMinimum[(CC - .02)^2, {b, 1}]*)
Plot3D[dd, {b, 0.0001, 3}, {a, 0.0001, 0.9999},
  PlotLabel -> "(dSN/dTc)/(dSN/dOD) Tt~Tb~Tc~300K",
  AxesLabel -> {"OD", " a", "dOD/dTc   "}, PlotPoints -> 25,
  PlotRange -> {0, .007}];
tcloud = 600;
Plot3D[dd, {b, 0.0001, 3}, {a, 0.0001, 0.9999},
  PlotLabel -> "(dSN/dTc)/(dSN/dOD) Tt~Tb~300,Tc~600",
  AxesLabel -> {"OD", " a", "dOD/dTc   "}, PlotPoints -> 25,
  PlotRange -> {0, .007}]
```

Program Darpaemit2
(*found plots are independent of g as expected for homogeneous illumination*)
g = 0.75;
tback = 300;
ttarg = 301;
p = a(1 + g)/2;
q = a(1 - g)/2;
z = Sqrt[(1 - a)(1 - a g)];
Tu = Exp[-b];
T = 2z/((1 - p + z)Exp[z b] - (1 - p - z)Exp[-z b]);
R = q(Exp[z b] - Exp[-z b])/((1 - p + z)Exp[z b] - (1 - p - z)Exp[-z b]);
CC = Tu(ttarg^4 - tback^4)/(Tu(ttarg^4 + tback^4) + 2 tback^4(R + T - Tu) +
        2(1 - R - T)tcloud^4);
dd = D[CC, tcloud]/D[CC, b];
b = 1;
(*FindMinimum[(CC - .02)^2, {b, 1}]*)
Plot3D[dd, {tcloud, 300, 600}, {a, 0.0001, 0.9999},
  PlotLabel -> "(dC/dTc)/(dC/dOD) OD=1,Tt~Tb~300K",
  AxesLabel -> {"Tc", " a", "dOD/dTc "}, PlotPoints -> 25,
  PlotRange -> {0, .015}];
b = 2;
Plot3D[dd, {tcloud, 300, 600}, {a, 0.0001, 0.9999},
  PlotLabel -> "(dC/dTc)/(dC/dOD) OD=2,Tt~Tb~300K",
  AxesLabel -> {"Tc", " a", "dOD/dTc "}, PlotPoints -> 25,
  PlotRange -> {0, .015}]

51

```
Program Darpaemit22
(*found plots are independent of g as expected for homogeneous illumination*)
g = 0.75;
tback = 300;
ttarg = 301;
p = a(1 + g)/2;
q = a(1 - g)/2;
z = Sqrt[(1 - a)(1 - a g)];
Tu = Exp[-b];
T = 2z/((1 - p + z)Exp[z b] - (1 - p - z)Exp[-z b]);
R = q(Exp[z b] - Exp[-z b])/((1 - p + z)Exp[z b] - (1 - p - z)Exp[-z b]);
CC = Tu(ttarg^4 -
        tback^4)/(Tu(ttarg^4 + tback^4) + 2 tback^4(R + T - Tu) +
        2(1 - R - T)tcloud^4 )^.5;
dd = D[CC, tcloud]/D[CC, b];
b = 1;
(*FindMinimum[(CC - .02)^2, {b, 1}]*)
Plot3D[dd, {tcloud, 300, 600}, {a, 0.0001, 0.9999},
  PlotLabel -> "(dSN/dTc)/(dSN/dOD) OD=1,Tt~Tb~300K",
  AxesLabel -> {"Tc", " a", "dOD/dTc   "}, PlotPoints -> 25,
  PlotRange -> {0, .007}];
b = 2;
(*FindMinimum[(CC - .02)^2, {b, 1}]*)
Plot3D[dd, {tcloud, 300, 600}, {a, 0.0001, 0.9999},
  PlotLabel -> "(dSN/dTc)/(dSN/dOD) OD=2,Tt~Tb~300K",
  AxesLabel -> {"Tc", " a", "dOD/dTc   "}, PlotPoints -> 25,
  PlotRange -> {0, .007}];
b = 3;
(*FindMinimum[(CC - .02)^2, {b, 1}]*)
Plot3D[dd, {tcloud, 300, 600}, {a, 0.0001, 0.9999},
  PlotLabel -> "(dSN/dTc)/(dSN/dOD) OD=3,Tt~Tb~300K",
  AxesLabel -> {"Tc", " a", "dOD/dTc   "}, PlotPoints -> 25,
  PlotRange -> {0, .007}]
```

```
Program Darpaemit3
(*found plots are independent of g as expected for homogeneous illumination*)
g = 0.75;
tback = 300;
ttarg = 301;
p = a(1 + g)/2;
q = a(1 - g)/2;
z = Sqrt[(1 - a)(1 - a g)];
Tu = Exp[-b];
T = 2z/((1 - p + z)Exp[z b] - (1 - p - z)Exp[-z b]);
R = q(Exp[z b] - Exp[-z b])/((1 - p + z)Exp[z b] - (1 - p - z)Exp[-z b]);
CC = Tu(ttarg^4 - tback^4)/(Tu(ttarg^4 + tback^4) + 2 tback^4(R + T - Tu) +
        2(1 - R - T)tcloud^4 );
dd = D[CC, tcloud]/D[CC, b];
a = .1;
(*FindMinimum[(CC - .02)^2, {b, 1}]*)
Plot3D[dd, {tcloud, 300, 600}, {b, 0, 3},
  PlotLabel -> "(dC/dTc)/(dC/dOD) a=0.1,Tt~Tb~300K",
  AxesLabel -> {"Tc", " OD", "dOD/dTc "}, PlotPoints -> 25,
  PlotRange -> {0, .015}];
a = .5;
Plot3D[dd, {tcloud, 300, 600}, {b, 0, 3},
  PlotLabel -> "(dC/dTc)/(dC/dOD) a=0.5,Tt~Tb~300K",
  AxesLabel -> {"Tc", " OD", "dOD/dTc "}, PlotPoints -> 25,
  PlotRange -> {0, .015}];
a = .9;
Plot3D[dd, {tcloud, 300, 600}, {b, 0, 3},
  PlotLabel -> "(dC/dTc)/(dC/dOD) a=0.9,Tt~Tb~300K",
  AxesLabel -> {"Tc", " OD", "dOD/dTc "}, PlotPoints -> 25,
  PlotRange -> {0, .015}]
```

```
Program Darpaemit33
(*found plots are independent of g as expected for homogeneous illumination*)
g = 0.75;
tback = 300;
ttarg = 301;
p = a(1 + g)/2;
q = a(1 - g)/2;
z = Sqrt[(1 - a)(1 - a g)];
Tu = Exp[-b];
T = 2z/((1 - p + z)Exp[z b] - (1 - p - z)Exp[-z b]);
R = q(Exp[z b] - Exp[-z b])/((1 - p + z)Exp[z b] - (1 - p - z)Exp[-z b]);
CC = Tu(ttarg^4 -
        tback^4)/(Tu(ttarg^4 + tback^4) + 2 tback^4(R + T - Tu) +
        2(1 - R - T)tcloud^4 )^.5;
dd = D[CC, tcloud]/D[CC, b];
a = 0.1;
(*FindMinimum[(CC - .02)^2, {b, 1}]*)
Plot3D[dd, {tcloud, 300, 600}, {b, 0, 3},
  PlotLabel -> "(dSN/dTc)/(dSN/dOD) a=0.1,Tt~Tb~300K",
  AxesLabel -> {"Tc", " OD", "dOD/dTc   "}, PlotPoints -> 25,
  PlotRange -> {0, .007}];
a = 0.5;
(*FindMinimum[(CC - .02)^2, {b, 1}]*)
Plot3D[dd, {tcloud, 300, 600}, {b, 0, 3},
  PlotLabel -> "(dSN/dTc)/(dSN/dOD) a=0.5,Tt~Tb~300K",
  AxesLabel -> {"Tc", " OD", "dOD/dTc   "}, PlotPoints -> 25,
  PlotRange -> {0, .007}];
a = 0.9;
(*FindMinimum[(CC - .02)^2, {b, 1}]*)
Plot3D[dd, {tcloud, 300, 600}, {b, 0, 3},
  PlotLabel -> "(dSN/dTc)/(dSN/dOD) a=0.9,Tt~Tb~300K",
  AxesLabel -> {"Tc", " OD", "dOD/dTc   "}, PlotPoints -> 25,
  PlotRange -> {0, .007}]
```

Program Darpaflare1
(*ir flare emission within a cloud is represented with a high temperature \
high albedo cloud where 1 -
    albedo is the absorption/
        emission which is just the volume fraction of the flare / (flare +
            white smoke) material multiplied by the ratio of flare absorption \
coefficient divided by smoke extinction coefficient ~
        radius smoke particles / Radius flare particles*)
g = 0.75;
p = a(1 + g)/2;
q = a(1 - g)/2;
z = Sqrt[(1 - a)(1 - a g)];
Tu = Exp[-b];
T = 2z/((1 - p + z)Exp[z b] - (1 - p - z)Exp[-z b]);
R = q(Exp[z b] - Exp[-z b])/((1 - p + z)Exp[z b] - (1 - p - z)Exp[-z b]);
(*Note that contrast transmittance is just contrast ratio -
    contrast with cloud/without cloud*)
CT = Tu(temptargovrbkg^4 + 1)/(Tu(temptargovrbkg^4 + 1) + 2(R + T - Tu) +
        2(1 - R - T)tempcloudovrbkg^4);
b = 1;
a = 1 - 10^ff;
temptargovrbkg = 1.0001;
Plot3D[CT, {tempcloudovrbkg, 2, 6}, {ff, -2, -5},
  PlotLabel -> "Flare Contrast Transmittance Tt~Tb,OD=1",
  AxesLabel -> {"Tf/Tb", "log rf/R", "CT "}, PlotPoints -> 25];
b = 1.5;
Plot3D[CT, {tempcloudovrbkg, 2, 6}, {ff, -2, -5},
  PlotLabel -> "Flare Contrast Transmittance Tt~Tb,OD=1.5",
  AxesLabel -> {"Tf/Tb", "log rf/R", "CT "}, PlotPoints -> 25];
b = 2;
Plot3D[CT, {tempcloudovrbkg, 2, 6}, {ff, -2, -5},
  PlotLabel -> "Flare Contrast Transmittance Tt~Tb,OD=2",
  AxesLabel -> {"Tf/Tb", "log rf/R", "CT "}, PlotPoints -> 25];
b = 2.5;
Plot3D[CT, {tempcloudovrbkg, 2, 6}, {ff, -2, -5},
  PlotLabel -> "Flare Contrast Transmittance Tt~Tb,OD=2.5",
  AxesLabel -> {"Tf/Tb", "log rf/R", "CT "}, PlotPoints -> 25]

Program Darpaflare2

(*ir flare emission within a cloud is represented with a high temperature \
high albedo cloud where 1 -
    albedo is the absorption/
        emission which is just the volume fraction of the flare / (flare +
            white smoke) material multiplied by the ratio of flare absorption \
coefficient divided by smoke extinction coefficient ~
        radius smoke particles / Radius flare particles*)

g = 0.75;
p = a(1 + g)/2;
q = a(1 - g)/2;
z = Sqrt[(1 - a)(1 - a g)];
Tu = Exp[-b];
T = 2z/((1 - p + z)Exp[z b] - (1 - p - z)Exp[-z b]);
R = q(Exp[z b] - Exp[-z b])/((1 - p + z)Exp[z b] - (1 - p - z)Exp[-z b]);
(*Note that contrast transmittance is just contrast ratio -
    contrast with cloud/without cloud*)
CT = Tu(temptargovrbkg^4 + 1)/(Tu(temptargovrbkg^4 + 1) + 2(R + T - Tu) +
        2(1 - R - T)tempcloudovrbkg^4);
SNT = (Tu CT)^.5;
b = 1;
a = 1 - 10^ff;
temptargovrbkg = 1.0001;
Plot3D[SNT, {tempcloudovrbkg, 2, 6}, {ff, -2, -5},
  PlotLabel -> "Flare S/N Transmittance Tt~Tb,OD=1",
  AxesLabel -> {"Tf/Tb", "log rf/R", "SNT "}, PlotPoints -> 25];
b = 1.5;
Plot3D[SNT, {tempcloudovrbkg, 2, 6}, {ff, -2, -5},
  PlotLabel -> "Flare S/N Transmittance Tt~Tb,OD=1.5",
  AxesLabel -> {"Tf/Tb", "log rf/R", "SNT "}, PlotPoints -> 25];
b = 2;
Plot3D[SNT, {tempcloudovrbkg, 2, 6}, {ff, -2, -5},
  PlotLabel -> "Flare S/N Transmittance Tt~Tb,OD=2",
  AxesLabel -> {"Tf/Tb", "log rf/R", "SNT "}, PlotPoints -> 25];
b = 2.5;
Plot3D[SNT, {tempcloudovrbkg, 2, 6}, {ff, -2, -5},
  PlotLabel -> "Flare S/N Transmittance Tt~Tb,OD=2.5",
  AxesLabel -> {"Tf/Tb", "log rf/R", "SNT "}, PlotPoints -> 25]

```
Program Darpaflare3
(*ir flare emission within a cloud is represented with a high temperature \
high albedo cloud where 1 -
    albedo is the absorption/
        emission which is just the volume fraction of the flare / (flare +
            white smoke) material multiplied by the ratio of flare absorption \
coefficient divided by smoke extinction coefficient ~
        radius smoke particles / Radius flare particles*)
g = 0.75;
p = a(1 + g)/2;
q = a(1 - g)/2;
z = Sqrt[(1 - a)(1 - a g)];
Tu = Exp[-b];
T = 2z/((1 - p + z)Exp[z b] - (1 - p - z)Exp[-z b]);
R = q(Exp[z b] - Exp[-z b])/((1 - p + z)Exp[z b] - (1 - p - z)Exp[-z b]);
(*Note that contrast transmittance is just contrast ratio -
    contrast with cloud/without cloud*)
CT = Tu(temptargovrbkg^4 + 1)/(Tu(temptargovrbkg^4 + 1) + 2(R + T - Tu) +
        2(1 - R - T)tempcloudovrbkg^4 );
tempcloudovrbkg = 4;
a = 1 - 10^ff;
temptargovrbkg = 1.0001;
Plot3D[CT, {b, 1, 2.5}, {ff, -2, -5},
  PlotLabel -> "Flare Contrast Transmittance Tt~Tb,Tf/Tb=4",
  AxesLabel -> {"OD", "log rf/R", "CT "}, PlotPoints -> 25];
tempcloudovrbkg = 5;
Plot3D[CT, {b, 1, 2.5}, {ff, -2, -5},
  PlotLabel -> "Flare Contrast Transmittance Tt~Tb,Tf/Tb=5",
  AxesLabel -> {"OD", "log rf/R", "CT "}, PlotPoints -> 25];
tempcloudovrbkg = 6;
Plot3D[CT, {b, 1, 2.5}, {ff, -2, -5},
  PlotLabel -> "Flare Contrast Transmittance Tt~Tb,Tf/Tb=6",
  AxesLabel -> {"OD", "log rf/R", "CT "}, PlotPoints -> 25];
tempcloudovrbkg = 3;
Plot3D[CT, {b, 1, 2.5}, {ff, -2, -5},
  PlotLabel -> "Flare Contrast Transmittance Tt~Tb,Tf/Tb=3",
  AxesLabel -> {"OD", "log rf/R", "CT "}, PlotPoints -> 25]
```

```
Program Darpaflare4
(*ir flare emission within a cloud is represented with a high temperature \
high albedo cloud where 1 -
    albedo is the absorption/
        emission which is just the volume fraction of the flare / (flare +
            white smoke) material multiplied by the ratio of flare absorption \
coefficient divided by smoke extinction coefficient ~
        radius smoke particles / Radius flare particles*)
g = 0.75;
p = a(1 + g)/2;
q = a(1 - g)/2;
z = Sqrt[(1 - a)(1 - a g)];
Tu = Exp[-b];
T = 2z/((1 - p + z)Exp[z b] - (1 - p - z)Exp[-z b]);
R = q(Exp[z b] - Exp[-z b])/((1 - p + z)Exp[z b] - (1 - p - z)Exp[-z b]);
(*Note that contrast transmittance is just contrast ratio -
    contrast with cloud/without cloud*)
CT = Tu(temptargovrbkg^4 + 1)/(Tu(temptargovrbkg^4 + 1) + 2(R + T - Tu) +
        2(1 - R - T)tempcloudovrbkg^4 );
SNT = (Tu CT)^.5;
tempcloudovrbkg = 4;
a = 1 - 10^ff;
temptargovrbkg = 1.0001;
Plot3D[SNT, {b, 1, 2.5}, {ff, -2, -5},
  PlotLabel -> "Flare S/N Transmittance Tt~Tb,Tf/Tb=4",
  AxesLabel -> {"OD", "log rf/R", "SNT "}, PlotPoints -> 25];
tempcloudovrbkg = 5;
a = 1 - 10^ff;
temptargovrbkg = 1.0001;
Plot3D[SNT, {b, 1, 2.5}, {ff, -2, -5},
  PlotLabel -> "Flare S/N Transmittance Tt~Tb,Tf/Tb=5",
  AxesLabel -> {"OD", "log rf/R", "SNT "}, PlotPoints -> 25];
tempcloudovrbkg = 6;
a = 1 - 10^ff;
temptargovrbkg = 1.0001;
Plot3D[SNT, {b, 1, 2.5}, {ff, -2, -5},
  PlotLabel -> "Flare S/N Transmittance Tt~Tb,Tf/Tb=6",
  AxesLabel -> {"OD", "log rf/R", "SNT "}, PlotPoints -> 25];
tempcloudovrbkg = 3;
a = 1 - 10^ff;
temptargovrbkg = 1.0001;
Plot3D[SNT, {b, 1, 2.5}, {ff, -2, -5},
  PlotLabel -> "Flare S/N Transmittance Tt~Tb,Tf/Tb=3",
  AxesLabel -> {"OD", "log rf/R", "SNT "}, PlotPoints -> 25]
```

Program Darpaflare5

(*ir flare emission within a cloud is represented with a high temperature \
high albedo cloud where 1 -
    albedo is the absorption/
        emission which is just the volume fraction of the flare / (flare +
            white smoke) material multiplied by the ratio of flare absorption \
coefficient divided by smoke extinction coefficient ~
        radius smoke particles / Radius flare particles*)

```
g = 0.75;
p = a(1 + g)/2;
q = a(1 - g)/2;
z = Sqrt[(1 - a)(1 - a g)];
Tu = Exp[-b];
T = 2z/((1 - p + z)Exp[z b] - (1 - p - z)Exp[-z b]);
R = q(Exp[z b] - Exp[-z b])/((1 - p + z)Exp[z b] - (1 - p - z)Exp[-z b]);
(*Note that contrast transmittance is just contrast ratio -
    contrast with cloud/without cloud*)
CT = Tu(temptargovrbkg^4 + 1)/(Tu(temptargovrbkg^4 + 1) + 2(R + T - Tu) +
        2(1 - R - T)tempcloudovrbkg^4 );
ff = -2;
a = 1 - 10^ff;
temptargovrbkg = 1.0001;
Plot3D[CT, {tempcloudovrbkg, 2, 6}, {b, 1, 2.5},
  PlotLabel -> "Flare Contrast Transmittance Tt~Tb,rf/R=.01",
   AxesLabel -> {"Tf/Tb", "OD", "CT "}, PlotPoints -> 25];
ff = -3;
a = 1 - 10^ff;
temptargovrbkg = 1.0001;
Plot3D[CT, {tempcloudovrbkg, 2, 6}, {b, 1, 2.5},
  PlotLabel -> "Flare Contrast Transmittance Tt~Tb,rf/R=.001",
   AxesLabel -> {"Tf/Tb", "OD", "CT "}, PlotPoints -> 25]
```

```
Program Darpaflare6
(*ir flare emission within a cloud is represented with a high temperature \
high albedo cloud where 1 -
    albedo is the absorption/
        emission which is just the volume fraction of the flare / (flare +
            white smoke) material multiplied by the ratio of flare absorption \
coefficient divided by smoke extinction coefficient ~
        radius smoke particles / Radius flare particles*)
g = 0.75;
p = a(1 + g)/2;
q = a(1 - g)/2;
z = Sqrt[(1 - a)(1 - a g)];
Tu = Exp[-b];
T = 2z/((1 - p + z)Exp[z b] - (1 - p - z)Exp[-z b]);
R = q(Exp[z b] - Exp[-z b])/((1 - p + z)Exp[z b] - (1 - p - z)Exp[-z b]);
(*Note that contrast transmittance is just contrast ratio -
    contrast with cloud/without cloud*)
CT = Tu(temptargovrbkg^4 + 1)/(Tu(temptargovrbkg^4 + 1) + 2(R + T - Tu) +
        2(1 - R - T)tempcloudovrbkg^4 );
SNT = (Tu CT)^.5;
ff = -2;
a = 1 - 10^ff;
temptargovrbkg = 1.0001;
Plot3D[SNT, {tempcloudovrbkg, 2, 6}, {b, 1, 2.5},
  PlotLabel -> "Flare S/N Transmittance Tt~Tb,rf/R=.01",
  AxesLabel -> {"Tf/Tb", "OD", "SNT "}, PlotPoints -> 25];
ff = -3;
a = 1 - 10^ff;
temptargovrbkg = 1.0001;
Plot3D[SNT, {tempcloudovrbkg, 2, 6}, {b, 1, 2.5},
  PlotLabel -> "Flare S/N Transmittance Tt~Tb,rf/R=.001",
  AxesLabel -> {"Tf/Tb", "OD", "SNT "}, PlotPoints -> 25]
```

60